LOGIC AND SET THEORY

Learning module description

GENERAL INFORMATION

1. Module title: Logic and set theory
2. Module code: DLOG LI0-E
3. Term: summer
4. Duration: 30h lectures + 30h exercises/laboratories
5. ECTS: 6
6. Module lecturer: Bartosz Naskręcki
7. E-mail: bartnas@amu.edu.pl
8. Language: English

DETAILED INFORMATION

1. Module aim is to familiarize students with basic notions of logic and set theory and its applications in computer science.
2. Pre-requisites in terms of knowledge, skills and social competences (where relevant):

SYLLABUS:

Week 1: Logical connectives, logical forms of sentences, tautologies of propositional logic. Truth tables. Formulas of the propositional calculus. Valuations. The value of the formula under the valuation. Definition of tautology of the propositional calculus. A list of important tautologies. Definition of logical equivalence and of satisfiability. Substitution in the propositional calculus.

Week 2: Logical rules of inference, deductive inferences. Inference and scheme of the inference. Definition of logical rule of inference. Relation between rules and tautologies.

Week 3: Conjunctive and disjunctive normal form, methods of transformation of formulas to a normal form, applications in verifying tautologies and satisfiability, applications to logical circuits.

Week 4: Quantifiers, propositional functions, terms and formulas, the laws of the predicate calculus, the laws of equality. Construction of an elementary language. Free and bound occurrences of variables. The notions of law (tautology) of the predicate calculus, of model of a set of formulas, and of logical inference. List of important laws and rules in the predicate calculus. Short equivalence proofs and implication proofs.

Week 5: Formal deductive systems. Information about Hilbert-style deduction system of the propositional calculus. The rules of a sequent system.

Week 6: Basic notions of set theory, the axiom of extensionality, set existence.The notion of set and relation between element and set. Ways of denoting of finite sets. Sets determined by a condition. Russell's antinomy. Axioms of extensionality, of empty set, pair, union and powerset.

Week 7: Operations on sets, inclusion, the laws of the algebra of sets, the derivation of the laws using the axiom of extensionality and laws of propositional calculus.

Week 8: Boolean algebras and the duality principle. Axioms of Boolean algebra for sets.

Week 9: Ordered pairs, Cartesian product and their definitions. Relations. Lemma of order pairs. Tuples and Cartesian product of sets. Many-argument relations. Properties of binary relations. Composition of relations, reverse relation. The laws of relation algebra.

Week 10: Functions, mappings, injection, surjection, bijection. Basic properties of functions. Images

and co-images. Indexed families of sets and infinite operations. Sequences. Relational structures. Homomorphism and isomorphism.
Week 11: Equivalence relations. Definitions of equivalence relations, equivalence classes and quotient set. Construction of integers and rationals.
Week 12: Ordering relations. Definitions of partial order and linear order. Hasse's diagram of a finite ordered set. Definitions of the smallest and largest element, minimal and maximal element, lower and upper bound, infimum, supremum. Construction of reals. Axiom of choice and its equivalents.
Week 13: Natural numbers and mathematical induction. Peano axioms. von Neumann construction. Mathematical induction principle. Minimum principle. Definitions of basic arithmetical operations.
Week 14: Cardinal numbers. Definition of a finite set. Comparing cardinal numbers. Cantor and Cantor-Bernstein theorem.
Week 15: Countable sets. Uncountable sets. Sets of cardinality continuum.